

# Is Scalar Synonymous with Real?

Activity developed at Cégep de Sherbrooke  
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In linear algebra, we talk about scalar product and product by a scalar, but use only real numbers as scalars. This activity is primarily aimed at studying the concept of scalar, therefore the concept of field, to ascertain that there are indeed scalars other than real numbers.

Based on the axiomatic definition of a field, students first build the field with two elements. They then learn about the field of complex numbers using the following approach: given the set  $K_{(\alpha,\beta)}$  consisting of all real pairs  $(\alpha, \beta)$  involving an addition and multiplication, verify that all the properties of a field are satisfied.

Afterwards, students must ascertain that the scalar product as defined in  $\mathbb{R}^2$  (and  $\mathbb{R}^n$ ) is not valid in  $K_{(\alpha,\beta)}$ . Based on the properties that define a scalar product, students obtain a generalization of the result.

This activity prompts students to do a more comprehensive self-study of certain elements of the *Linear Algebra* course. This learning approach is based on a series of questions that compels students to make connections with theoretical concepts shown in class. Each question prepares the next one, either by its result, either by the method used to answer the question, or by the mathematical object being studied.

This activity is original in that it more naturally introduces imaginary numbers by the properties of operations in a field, rather than by the artificial definition: a number which, when squared, equals -1. Its originality also lies in the fact that students work on complex numbers throughout almost the entire semester without these numbers ever being named.