

Activity **11**



Is Scalar Synonymous with Real?

Activity developed at Cégep de Sherbrooke
By **MARIE-JANE HAGUEL, NICOLAS PFISTER**
and **SYLVIE SAVAGE**

Is Scalar Synonymous with Real?

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Author's Name

Marie-Jane Haguel, Nicolas Pfister and Sylvie Savage

Originating Cégep

Cégep de Sherbrooke

Author's E-Mail Address

**Mijoh@allstream.net
Nicolas.pfister@cegepsherbrooke.qc.ca
Sylvie.savage@cegepsherbrooke.qc.ca**

Scientific Discipline

Mathematics

Average Age of Students

18-19 years old

Course Title and Number

Linear Algebra (201-NYC-05)

Duration of Activity

6 weeks

NOTE

In this document, the masculine is used without discrimination and solely to make the text easier to read.

Appendices are available in PDF and Word format on the CD provided with this document.

In addition, an instructional analysis of the activity is available in the pedagogical treasures section (*Trésors pédagogiques*) on the Saut Quantique Web site at:

<http://www.apsq.org/sautquantique>.

Use of this text is authorized for instructional purposes, provided that author's name and college are mentioned.

Adherence to these recommendations will encourage authors to share their experience.



Is Scalar Synonymous with Real?

Description of Activity

OVERVIEW

In linear algebra, we talk about scalar product and product by a scalar, but use only real numbers as scalars. This activity is primarily aimed at studying the concept of scalar, therefore the concept of field, to ascertain that there are indeed scalars other than real numbers.

Based on the axiomatic definition of a field, students first build the field with two elements. They then learn about the field of complex numbers using the following approach: given the set $K_{(\alpha,\beta)}$ consisting of all real pairs (α,β) involving an addition and multiplication, verify that all the properties of a field are satisfied.

Afterwards, students must ascertain that the scalar product as defined in \mathbb{R}^2 (and \mathbb{R}^n) is not valid in $K_{(\alpha,\beta)}$. Based on the properties that define a scalar product, students obtain a generalization of the result.

RELEVANCE AND ORIGINALITY OF ACTIVITY

This activity prompts students to do a more comprehensive self-study of certain elements of the *Linear Algebra* course. This learning approach is based on a series of questions that compels students to make connections with theoretical concepts shown in class. Each question prepares the next one, either by its result, either by the method used to answer the question, or by the mathematical object being studied.

This activity is original in that it more naturally introduces imaginary numbers by the properties of operations in a field, rather than by the artificial definition: a number which, when

squared, equals -1. Its originality also lies in the fact that students work on complex numbers throughout almost the entire semester without these numbers ever being named.

Objectives and Relation to the Program

PEDAGOGICAL OBJECTIVES OR TARGETED COMPETENCIES

Plan and organize one's work and schedule, to meet specific deadlines.

Make an accurate judgement on results obtained. Realize that it is impossible to grasp the scope of a question until it has been answered.

Understand what an operation is. The concept of operation is often limited to knowing which key to press on the calculator. This assignment brings students to explore and understand the concept of operation by putting them in a context that is sufficiently unfamiliar to compel them to use the definition of the operation and its associated properties.

Realize the importance of the concept of closure involved in the notions of vector space and field, and all mathematical structures.

Learn to use algebra manipulations in a framework featuring several different levels of abstraction: as vectors are defined on the field of complex numbers, the components of these vectors have components of their own.

Understand the concept of generalization in mathematics and learn to obtain a generalization of a result when its scope is extended.

LINK BETWEEN THE ACTIVITY AND THE PROGRAM

General Program Goals Targeted

This activity targets the following general goals of the *Science* program:

- To reason logically;
- To communicate effectively;
- To learn autonomously;
- To work as members of a team;
- To apply what they have learned to new situations.

Link with Course

This activity recaps, in a broad framework, the properties studied in the course for geometric and algebraic vectors using real numbers, as well as the concept of scalar product.

Link with Other Courses

In *Differential Calculus* and *Integral Calculus* courses, students use the properties of real numbers, which are outlined in this assignment.

These calculus courses also feature the concept of closure for operations performed on functions.

Number of Students and Educational Support

APPROXIMATE NUMBER OF STUDENTS IN CLASS

30 students

NUMBER OF STUDENTS PER TEAM

2 people

EDUCATIONAL SUPPORT

The teacher schedules some time in class for this activity. He is also available outside of class hours to provide student assistance.

The activity is divided into three parts, each followed by a feedback session in class to ensure that each student correctly understands the concepts and is able to reinvest his learnings in the next part.

Conducting the Activity

CONDUCTING THE ACTIVITY AND TIME REQUIRED TO COMPLETE EACH STEP

Before

The concepts of adding vectors, multiplying by a scalar, vector space and scalar product are explained in class.

During

The activity includes three assignments (see Appendices S.1, S.2 and S.3), where students pair up and work mostly outside of class hours.

The material to be studied during the semester is divided into three parts, each ending with an exam. The work is done concurrently with the material learned in class. Students have two weeks during each of the three parts of the course, to complete the three assignments.

The teacher corrects the assignments and returns them to the students before the next part.

After

Students are invited to read the chapter on complex numbers in Vincent Papillon's textbook (see the Media Directory), and compare the approach presented in this activity with the more traditional approach found in the book.



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The teacher can also choose to ask a short question on this activity during the exam.

Students derive a great deal of satisfaction from the work they accomplish.

Evaluation and Required Material

SUGGESTED EVALUATIONS

In the fall of 2004, this assignment was worth 10% of the final grade. A weighting of 15% would be more appropriate to represent more accurately the amount of work done by students.

The correction grid for each assignment is included as an appendix.

APPENDICES

Teacher

Appendix T.1: Correction Grid for the First Assignment

Appendix T.2: Correction Grid for the Second Assignment

Appendix T.3: Correction Grid for the Third Assignment

Students

Appendix S.1: First Assignment

Appendix S.2: Second Assignment

Appendix S.3: Third Assignment

Note:

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Other Suggestions and Media Directory

OTHER IDEAS TO EXPLORE

If \vec{u} and \vec{v} are vectors of a vector space \mathbb{R} , then the Pythagorean Theorem states that:

$$\|\vec{u}\|^2 + \|\vec{v}\|^2 = \|\vec{u} + \vec{v}\|^2$$

which, in scalar product notation, can be written as follows:

$$\vec{u} \cdot \vec{u} + \vec{v} \cdot \vec{v} = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}).$$

It would be interesting to see how the Pythagorean Theorem generalizes to a vector space with complex numbers, using the scalar product defined on these complex numbers.

MEDIA DIRECTORY

PAPILLON, Vincent (1993). Vecteurs, matrices et nombres complexes, Mont-Royal, Modulo Éditeur, 387 p.